Mathematics to Ponder While the Wave Passes

In the last *SUNY Broome Today*, I posted a sightly techy study on how all epidemics can be modelled by mathematics. The model was first invented by Pierre Verhulst and others soon after him, in the 1830's.

Would you pour a hot cup of coffee, and ponder two questions?

(1) The Verhulst (or logistic) model is an equation stemming from a differential equation. How real is all this? Not the applications, they are based on data, which is expected to be real in the sense of not being erroneous. How real is the Verhulst equation? Is it entirely a construction in our minds, after studying Verhulst, or is it part of something more permanent? Another way of putting it is, is the equation invented, or discovered?

A barker grabs a microphone suspended from the ceiling. "*Laaadies and gentlemen!* For your entertainment tonight, in *this* corner, weighing in at about 2400 years of age, the **materialists**, with their trainers, the world champions Democritus and Epicurus..." They say that the equation, and all of mathematics, is invented, and has no permanent stance. If man disappears, then so does all of math (as opposed to the Pyramids, which still have some centuries left in them). What we hear, feel, see and touch **is** actually out there, although its actual essence may be shielded from our minds by many layers of processing (we perceive the world better than an autonomous vehicle does, but soon, they will be better than us at it).

"*Aaaand* in the *opposite* corner, weighing in at 2300 years of age, the **idealists**, with their trainer, the magnificent Plato the Athenian..." They say that all of nature is "visible intelligence." The universe is already out there, and we just turn our beacon of perception this way and that, and gather ideas, like the Verhulst equation. If man disappears, mathematics still remains unaltered, perhaps to be rediscovered by some civilization from another galaxy. What we hear, feel, see and touch **may** be out there, but we are forever shielded from its actual essence since our minds relentlessly process sense data until it becomes us (we emphatically cannot perceive the world as an autonomous vehicle does).

Who will win this match? They've been duking it out for centuries, and it's still a tie. The barker yells, "Now, picks yer choice, and pays yer money!" Some folks don't bet at all, but they haven't quite absorbed the spirit of philosophy... yet.

(2) *Prologue:* Some human endeavors are permanent, but not cumulative, e.g., Shakespeare's works, or the Pyramids. Some are cumulative, but not permanent, e.g., theories in physics or chemistry. Chemistry has a particularly interesting history in this regard. When alchemy was found to be pretty much a will-o'-the-wisp, something called the "phlogiston theory" of heat was conceived. When it was insufficient to explain new experiments, it was replaced by the "caloric theory" of heat. Sadie Carnot, one of the founders of thermodynamics, held that heat was an fluid called *caloric*: something imponderable that moved from hot objects to cold ones. (Imponderable: sophisticated term meaning, "I don't really know what the heck I'm talking about.") This theory in turn became archaic. The industrial revolution needed a real explanation for why engines worked, or didn't. Finally, this part of chemistry was welded to physics in the modern theory of thermodynamics (a Greek amalgam meaning "heat-motion"), and that's how it is today, even explaining why your car runs. The steps were accomplished by cumulative observation, but the breaks from the old were decisive. Physicist Thomas Kuhn calls these changes "paradigm shifts".

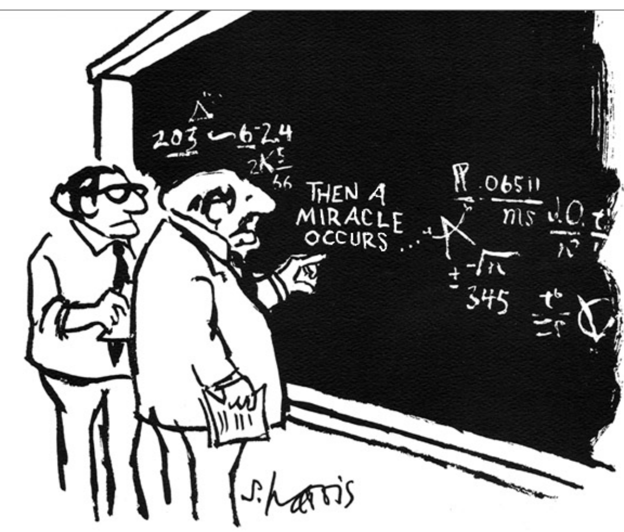
Question: Mathematics has the unique property of being **both** permanent and cumulative. Why is that? The Verhulst equation is valid today, 180 years later. The Pythagorean Theorem is perfect also, and it is 2500 years old. This theorem is perhaps the most fruitful theorem ever discovered (or invented, per the fight in Question 1). Why are these things as close to absolute truth as humanists can hold? (God is another thing altogether.)

These theorems are so correct because they have run the gauntlet of deductive proof. What is that? Well, anyone who has gone through high school geometry learned some of it, and I imagine that you loved it, or detested it. Remember the arduous process of deducing conclusions from previously established premises? That was deductive proof. It was like solving a detective's case, but completely airtight, with no doubts at all. It was so perfect that when we students found an error in the textbook, it wasn't our opinion against the author's (what chutzpah!). Here, we were absolutely correct, and the book was completely wrong. We felt like conquerors. In contrast, we could never tell a history teacher that a certain essay was wrong – we weren't PhD's in the subject.

Thus, permanence comes from deductive proof, in the sense established by logic. Deduction was the fortress that Greek (and Chinese, and Indian) mathematicians were driven to under the withering attacks from philosophers, such as Zeno of Elea. Deduction was unassailable. In this form of argumentation, if one denies the conclusion, then one has denied the premises. But the premises had **already** been accepted by the arguer, in order to proceed. Checkmate. Utter checkmate. The conclusion must be accepted, too. So, the Pythagorean theorem and all the rest of mathematics moved forward like an avalanche rolling on pure logic. Mathematics is therefore cumulative, and permanent.

Try your hand at a deductive proof. Prove that for *any* prime number starting at 5, it can be written as $6n \pm 1$, where n is some whole number. For instance, $5 = 6(1) - 1$, so the number we seek is $n = 1$. Again, for the prime 61, $61 = 6(10) + 1$, so $n = 10$ in this case. Be careful, for while it is true that $25 = 6(4) + 1$, this is irrelevant, because 25 is not a prime. Another hint. List the whole numbers like this:

1 2 3 4 5 **6** 7 8 9 10 11 **12** 13 14 15 16 17 **18** 19 20 21 22 23 **24** 25 ... and see what you can find.



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."