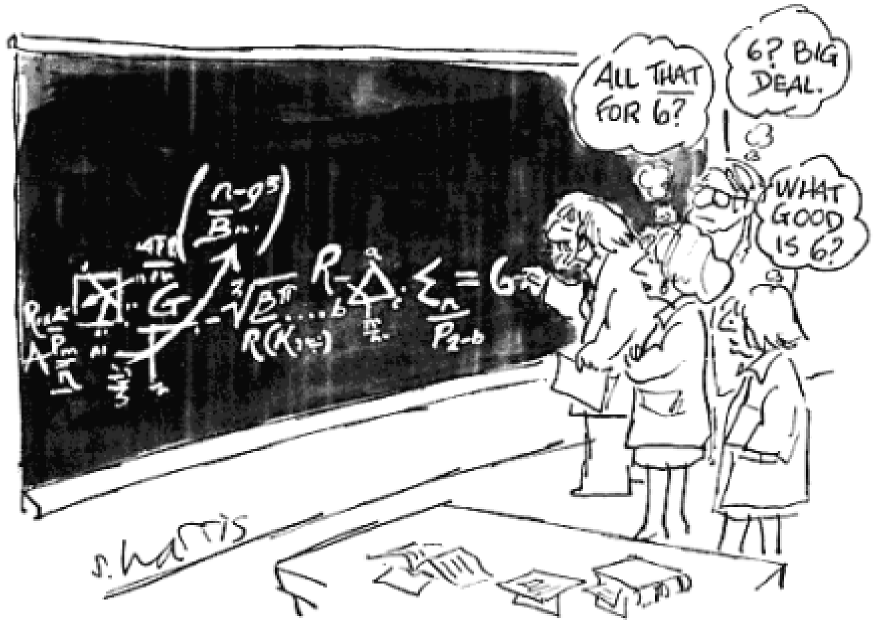

From the last column, were you to able to prove that all primes starting at 5 can be expressed as $6n \pm 1$, for some whole number n that depends on the prime being considered? The proof is at the end of today's column.

What is Mathematics?

This is one of those fundamental questions for all STEM people, at the very least. There are more difficult questions, such as, "What is beauty?", but leave that to the philosophers. For our purposes here, a utilitarian reply is, "it is the field that supplies all the kinds of numbers, with their logical structures, that civilizations need." It's a good starting point, for all else follows. We begin the story of numbers as far back as can be imagined.

The whole numbers, 1, 2, 3, … come to us from prehistory. Tally marks carved into bones predate numbers, but carry the germ of the idea, that is, a one-to-one correspondence between the tally and whatever was being counted.

Some archaeologists make the argument that number symbols predate alphabetic letters. The process outlined here takes place in ancient Sumer, in 8000 BC. Suppose a herd of 50 goats was to be sold to a buyer, far away. To verify this number, 50 little clay marbles (or maybe goat shapes) would be wrapped in a clay cover, and sealed by baking it. This was a "bulla." If the bulla arrived broken, then it would be assumed that someone had tampered with the number of marbles in it, and the transaction was suspect. Otherwise, 50 marbles would match 50 goats, and all was honest.

Soon enough, each bulla would carry the seal of the seller and of the buyer, and a symbol for "50" would also be imprinted. This wasn't just good bookkeeping, this was the first use of an abstract symbol instead of 50 tally marks (and then, who needed the 50 marbles anymore!). The merchants would soon realize that using five marks of ten each would convey the idea of fifty, so that a small set of symbols could be chained together in order to symbolize large quantities. Thus it was that numbers may have preceded alphabets as written communication. This is still a theory, although many bullae have been unearthed. And we do see the close connection with Roman numerals and our fingers as tally markers: I, II, III, IIII, V. The "V" would obviously be the symbol for a hand.

But there were limitations to these simple numbering systems. One of them was that it was not possible to write large numbers, and this influenced what was thought to be uncountable. For instance, there was no standard Roman numeral for one million. In Genesis 22:17, God says to Moses, "I promise that I will give you as many descendants as there are stars in the sky or grains of sand along the seashore." Such numbers were not expressible in those days; hence, those amounts were uncountable, easily confused with the infinite. ("Why is there no number for it? Because it's infinite.")

Enter the astonishing intellect of Archimedes, flourishing about 230 BC in Syracuse, Sicily. He wrote a treatise invitingly called *The Sand Reckoner*, in which he took the Greek numbering system (which was as primitive as the Roman's) and put it on steroids. He began with the largest named number, a myriad, or $10,000$. Then he defined a "myriadmyriad," literally a myriad myriads, or $100,000,000$, or 100 million. He called this a unit of the second order. Next, a 2nd order of 2nd orders would be a unit of the third order:
$100,000,000 \times 100,000,000 = 10,000,000,000,000,000$, or 10 quadrillion. This is inconceivable to people even today, but not to Archimedes. And he didn't stop there...

Building up higher orders, he was able to estimate the number of grains of sand that would fill the Earth! Let's do that. We can imagine a grain of sand as a tiny cube with a volume of 0.1 cubic millimeter. At that size, it will take 10 billion grains to fill a cubic meter. (Archimedes would say, "Easy! That's 100 units of the second order.")

We now need some modern notation: powers of ten. We know that $10^4 = 10,000$, and $10^8 = 100,000,000$, the 2nd order unit. In short, the exponent gives the number of zeros in today's system. Thus, 10 billion is 10^{10} (10 zeros there), the grains of sand per cubic meter. Now, the Earth has a volume of about 10^{21} cubic meters. Archimedes knew this because of three facts. First, the astronomer Eratosthenes, a friend of Archimedes, had calculated the circumference of the Earth, and the value obtained was very near to the true value. Second, the Greeks knew that the Earth was a sphere. Third, Archimedes himself had discovered the formula for the volume of a sphere - another reason why he was the most brilliant mathematician of the ancient world. Therefore, it takes about $10^{10} \times 10^{21} = 10^{31}$ grains of sand to fill the entire world. Today, that would be called 10 nonillions (no, no, not a "gazillion"). Archimedes would call this one tenth of a unit of the 3rd order. The sand reckoner, for sure!

Incidentally, we have just used one of the laws of exponents, in $10^{10} \times 10^{21} = 10^{31}$. When the base, 10, is the same, we multiply by adding exponents. Archimedes discovered this law.

Proof.

Pick any whole number greater than 1, and substitute it in the table wherever you see *n*. The list and comments will repeat forever downward, beyond $6n + 6$:

…	…	…
$6n - 5$	$= 6(n-1)+1$	odd like $6n+1$, maybe prime, maybe not
$6n - 4$	$= 6(n-1)+2$	even like $6n-2$, not prime
$6n - 3$	multiple of 3	not prime
$6n - 2$	even number	not prime
$6n - 1$	odd number	maybe prime, maybe not
$6n$	multiple of 6	not prime
$6n + 1$	odd number	maybe prime, maybe not
$6n + 2$	even number	not prime
$6n + 3$	multiple of 3	not prime, like $6n-3$
$6n + 4$	$= 6(n+1)-2$	even like $6n-2$, not prime
$6n + 5$	$= 6(n+1)-1$	odd like $6n-1$, maybe prime, maybe not
$6n + 6$	$= 6(n+1)$	like $6n$, a multiple of 6, not prime
…	…	…

Now comes the killer move: Look at he numbers $6n + 1$ and $6n - 1$. They are definitely odd, and may or may not be prime, depending on *n*. Therefore, **all** primes starting at 5 must fall into either the $6n + 1$ or $6n - 1$ category, depending on the prime. Checkmate, and proof.